EXERCISE – V

JEE PROBLEMS

1. (a) The triangle PQR is inscribed in the circle, $x^2+y^2=25$. If Q and R have co-ordinates (3, 4) & (–4, 3) respectively, then $\angle QPR$ is equal to **[JEE 2000(Scr.), 1 + 1]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

(b) If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ & $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then 'k' is

(A) 2 or $-\frac{3}{2}$ (B) – 2 or $-\frac{3}{2}$

(C) 2 or $\frac{3}{2}$ (D) – 2 or $\frac{3}{2}$

2. (a) Extremities of a diagonal of a rectangle are (0, 0) & (4, 3). Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal. **[REE 2000(Mains), 3+3+5]**

(b) Find the point on the straight line, $y = 2x + 11$ which is nearest to the circle, $16(x^2 + y^2) + 32x – 8y – 50 = 0$.

(c) A circle of radius 2 units rolls on the outerside of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles is inclined at an angle of 60º with x-axis.

3. (a) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals. **[JEE 2001 (Scr.), 1]**

(A) $\sqrt{PQ.RS}$ (B) $\frac{PQ+RS}{2}$

(C) $\frac{2PQ.RS}{PQ+RS}$ (D) $\sqrt{\frac{(PQ)^2+(RS)^2}{2}}$

(b) Let $2x^2 + y^2 – 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[JEE 2001 (Mains), 5]**

4. (a) Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 – 2x – 6y + 6 = 0$ and $x^2 + y^2 + 2x – 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. **[REE 2001 (Mains), 3]**

(b) Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. **[REE 2001 (Mains), 5]**

5. (a) If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x – 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

(b) If $a > 2b > 0$ then the positive value of m for which $y = mx – b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x – a)^2 + y^2 = b^2$ is **[JEE 2002 (Scr.), 3+3]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

6. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 – 2x – 6y + 6 = 0$ **[JEE 2004(Scr.)]**

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$

7. Line $2x + 3y + 1 = 0$ is a tangent to a circle at (1, –1). This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points (0, –1) and (–2, 3). Find equation of circle. **[JEE 2004, 4]**

8. A circle is given by $x^2 + (y – 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is **[JEE 2005 (Scr.)]**

(A) $\{(x,y) : x^2 = 4y\} \cup \{(x,y) : y \le 0\}$

(B) $\{(x,y) : x^2 + (y-1)^2 = 4\} \cup \{(x,y) : y \le 0\}$

(C) $\{(x,y) : x^2 = y\} \cup \{(0,y) : y \le 0\}$

(D) $\{(x,y) : x^2 = 4y\} \cup \{(0,y) : y \le 0\}$

9. (a) Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is **[JEE 2007, 3 + 3]**

(A) 3 (B) 2 (C) 3/2 (D) 1

(b) Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.
Statement-I : The tangents are mutually perpendicular.
because
Statement-II : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.
(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I
(B) Statement-I is true, statement-II is true; statement-II is **NOT** correct explanation for statement-I
(C) Statement-I is true, Statement-II is False
(D) Statement-I is False, Statement-II is True

10. (a) Consider the two curves $C_1 : y^2 = 4x$; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then, **[JEE 2008, 3 + 3 + 4 + 4 + 4]**
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

(b) Consider, $L_1 : 2x + 3y + p - 3 = 0$;
$L_2 : 2x + 3y + p + 3 = 0$,
where p is a real number, and $C : x^2+y^2+6x-10y+30=0$
Statement-I : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.
and
Statement-II : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.
(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I
(B) Statement-I is true, statement-II is true; statement-II is **NOT** correct explanation for statement-I
(C) Statement-I is true, Statement-II is False
(D) Statement-I is False, Statement-II is True

(c) Comprehension (3 questions together) :
A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP and D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.
(i) The equation of circle C is

(A) $(x-2\sqrt{3})^2 + (y-1)^2 = 1$ (B) $(x-2\sqrt{3})^2 + (y+\frac{1}{2})^2 = 1$

(C) $(x-\sqrt{3})^2 + (y+1)^2 = 1$ (D) $(x-\sqrt{3})^2 + (y-1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, \quad y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, \ y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x, y = 0$

11. (a) Tangents drawn from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$
touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is
(A) $x^2 + y^2 + 4x - 6y + 19 = 0$
(B) $x^2 + y^2 - 4x - 10y + 19 = 0$
(C) $x^2 + y^2 - 2x + 6y - 29 = 0$
(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

(b) The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is **[JEE 2009, 3 + 4]**

12. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is **[JEE 2010]**
{**Note :** [k] denotes the largest integer less than or equal to k}
13. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point **[JEE 2011]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$

(C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

14. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If **[JEE 2011]**

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

15. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is **[JEE 2012]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$

(B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$

(D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question Nos. 16 to 17

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3}, 1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[JEE 2012]**

16. A possible equation of L is

(A) $x - \sqrt{3}\,y = 1$ (B) $x + \sqrt{3}\,y = 1$

(C) $x - \sqrt{3}\,y = -1$ (D) $x + \sqrt{3}\,y = 5$

17. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$

(C) $x + \sqrt{3}\,y = 4$ (D) $x + 2\sqrt{2}\,y = 6$